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# SUBMERSIONS FROM ALMOST PARA-HERMITIAN MANIFOLDS

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*Dedicated to my father.*

Abstract. In this paper we study about anti-holomorphic semi-invariant submersion from almost para-Hermitian manifolds. We give example and investigate integrability of all distribution involved in the submersion also we prove that the O'Niell's tensor T vanishes on the invariant vertical distribution. We give necessary and sufficient condition for totally geodesicness and harmonicity of such type of submersions.

## 1. Introduction

The theory of Riemannian submersions was introduced by O'Niell and Gray in [4], [1], respectively and Watson introduced the Riemannian submersions between almost complex manifolds in [8]. Riemannian submersion between almost contact manifolds were studied by Chinea in [9] under the name of almost contact submersion. Riemannian submersion have been also studied for quaternionic Kähler manifolds [20] and para-quaternionic Kähler manifolds [3], [21]. Most of the studies related to Riemannian or almost Hermitian submersions can be found in book [16]. The study of anti-invariant Riemannian submersions from almost Hermitian manifolds were introduced by Sahin [6]. Recently Sahid and Tanveer extended this notion to the case when the total manifold is nearly Kähler in [2]. There are some recent paper which involve other structures such as Lagrangian submersion [10], almost product submersion [22], sasakian [11], anti invariant Riemannian submersion [19], semi-invariant submersion [7], semi-slant submersion [14] and H-slant submersion [13]. On the other hand para-complex manifolds, almost para-Hermitian manifolds and para-Kähler manifolds were defined by Libermann [18] in 1952. In fact such manifolds arose in [17].

Semi-Riemannian submersion were initiated by O'Niell in his book [5]. It is well known that such submersion have their application in Kaluza Klien theories, Yang Mills equation, string theories and supergravity. For application of semi-Riemannian submersion, see[15]. Since almost para-Hermitian manifolds are semi-Riemannian manifolds so we can consider a semi-Riemannian submersion from semi-Riemannian manifolds. In particular, the notion of semi-invariant is a natural generalization of the notion anti-invariant submersion.

The paper is organized as follows. In section 2, we give some notions needed for the paper. In section 3, we give the definition of anti-holomorphic semi-invariant

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semi-Riemannian submersions provide an example. In section 4, we study the integrability and totally geodesicness of the distributions involved in the definition of anti-holomorphic semi-invariant semi-Riemannian submersion. In section 5, we shall prove the totally geodesicness and harmonicity of an anti-holomorphic semi-invariant semi-Riemannian submersion.

## 2. Preliminaries

In this section, we define almost para-Hermitian manifolds recall that the notion of semi-Riemannian submersions from semi-Riemannian manifolds and give a brief review of semi-Riemannian submersions also we define an anti-holomorphic semi-invariant semi-Riemannian submersion.

An almost para-Hermitian manifold is a manifold $M$ endowed with an almost para-complex structure $J \neq \pm I$ and a semi-Riemannian metric $g$ such that

$$J^2 = I \tag{2.1}$$

$$g(JX, JY) = g(X, Y) \tag{2.2}$$

for $X, Y \in \Gamma TM$, where $I$ is the identity map the dimension of $M$ is even and the signature of $q$ is $(m, m)$, where $\dim M = 2m$. Consider an almost para-Hermitian manifold $(M, J, g)$ and denoted by $\nabla$ the Livi-Civita connection on $M$ with respect to $g$. Then $M$ is called a para-Kähler manifold if $J$ is parallel with respect to $\nabla$ that is

$$(\nabla_X J)Y = 0 \tag{2.3}$$

for $X, Y \in \Gamma TM$ [19].

Let $(M, g)$ and $(N, g_n)$ be two connected semi-Riemannian manifolds of index $r$ $(0 \leq r \leq \dim M)$ and $\acute{r}$ $(0 \leq \acute{r} \leq \dim N)$ respectively, with $r > \acute{r}$. A semi-Riemannian submersion is a smooth map $\pi : M \to N$ which is onto and satisfies the following conditions:

(a) $\pi_{*x} : T_xM \to T_{\pi(x)}N$ is onto for all $x \in M$.

(b) The fibers $\pi^{-1}(\acute{x})$, $\acute{x} \in N$ are semi-Riemannian submanifolds of $M$.

(c) $\pi_*$ preserves scaler product of vectors normal to fibers.

The vectors tangent to the fibers are called vertical and those normal to the fibers are called horizontal. We denote by $D^\perp$ and $D$ the vertical distribution and the horizontal distribution respectively. Also by $v$ and $h$ the vertical and horizontal projection respectively. A horizontal vector field $X$ on $M$ is said to be basic if $X$ is $\pi$ related to a vector field $\acute{X}$on $N$ has unique horizontal lift $X$ to $M$ and $X$ is basic.

We recall that the section of $D^\perp$ and $D$ are called the vertical vector fields and horizontal vector fields respectively. A semi-Riemannian submersion $\pi : M \to N$ determines two $(1, 2)$ tensor $T$ and $A$ on $M$, by the formula

$$\begin{aligned} T_E F &= h\nabla_{vE} vF + v\nabla_{vE} hF \\ A_E F &= h\nabla_{hE} vF + v\nabla_{hE} hF \end{aligned} \tag{2.4}$$

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for any $E,F \in \Gamma(TM)$, where $v$ and $h$ are vertical and horizontal projection. From (9) it is easy to see that $T_E$ and $A_E$ are skew symmetric operators on the tangent bundle of $M$ reversing the vertical and horizontal distributions. We summarize the properties of the tensor fields $T$ and $A$. Let $X, Y$ be vertical and $U, W$ be horizontal vector fields on $M$, then we have the following results

$$T_U V = T_V U \tag{2.5}$$

$$A_X Y = -A_Y X = \frac{1}{2} v[X, Y] \tag{2.6}$$

On the other hand from (4) we get

$$\begin{aligned} \nabla_U W &= T_U W + \hat{\nabla}_U W && (2.9) \\ \nabla_U X &= T_U X + h(\nabla_U X) && (2.8) \\ \nabla_X U &= A_X U + v(\nabla_X U) && (2.9) \\ \nabla_X Y &= A_X Y + h(\nabla_X Y) && (2.10) \end{aligned}$$

where $\hat{\nabla}_U W = v\nabla_U W$ and $h(\nabla_U X) = h(\nabla_X U) = A_X U$ if $X$ is basic. It is easy to observe that $T$ acts on the fibers as the second fundamental form while $A$ acts on the horizontal distribution and measure of the obstruction to the integrability of the distribution.

Finally, we recall that the notion of second fundamental form of a map between semi-Riemannian manifolds. Let $(M, g)$ and $(N, g_n)$ be semi-Riemannian manifolds and $\phi : (M, g) \to (N, g_n)$ a smooth map. Then the second fundamental form of $\phi$ is given by

$$(\nabla \phi_*)(X, Y) = \nabla^{\phi}_X \phi_* Y - \phi_*(X, Y) \tag{2.11}$$

for $X, Y \in \Gamma TM$. Where $\nabla^{\phi}$ is the pullback connection and we denote conveniently by $\nabla$ the Livi-Civita connection of the metric $g$ and $g_n$ recall that the $\phi$ is said to be harmonic if $trace(\nabla \phi_*) = 0$ and $\phi$ is called a totally geodesic map if $(\nabla \phi_*)(X, Y) = 0$ for $X, Y \in \Gamma TM$. It is known that second fundamental form is symmetric.

## 3. Anti-Holomorphic Semi-invariant submersion

**Definition 1.** *Let $M$ be a 2k -dimensional almost para-Hermitian manifold with Hermitian metric $g$ and almost complex structure $J$ and $N$ be the semi-Riemannian metric $g_n$. A semi-Riemannian submersion $\pi : (M, g, J) \to (N, g_n)$ is called semi-invariant submersion if there is a distribution $D \subset \ker \pi_*$ such that*

$$\ker \pi_* = D \oplus D^{\perp}, \; J(D) = D, \quad J(D^{\perp}) \subset (\ker \pi_*)^{\perp}$$

where $D^{\perp}$ is the orthogonal complement of $D$ in $\ker \pi_*$.

In this case the horizontal distribution $(\ker \pi_*)^{\perp}$ is decomposed as

$$(\ker \pi_*)^{\perp} = J(D^{\perp}) \oplus \mu$$

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where $\mu$ is the orthogonal complementary distribution of $J(D^{\perp})$ in $(\ker \pi_*)^{\perp}$ and it is invariant with respect to $J$.

**Definition 2.** *Let $\pi : (M, g, J) \to (N, g_n)$ be a semi-invariant semi-Riemannian submersion then we said $\pi$ is an anti-holomorphic semi-invariant semi-Riemannian submersion if*

$$(\ker \pi_*)^{\perp} = J(D^{\perp}) \quad \text{i.e. } \mu = \{0\}$$

If we let the dimension of distribution $D$ (resp. $D^{\perp}$) is $2m$ (resp. $2n$). Then the $\dim(M) = 2m + 2n$ and $\dim(N) = n$.

An anti-holomorphic semi-invariant semi-Riemannian submersion is called a proper anti-holomorphic semi-invariant semi-Riemannian submersion if $m$ and $n$ are non zero.

Now we are ready to study anti-holomorphic semi-invariant semi-Riemannian submersion from para-Kähler manifold. We get how the Kählerian structure on $M$ places restriction on the tensor fields $T$ and $A$ of an anti-holomorphic semi-invariant semi-Riemannian submersion $\pi : (M, g, J) \to (N, g_n)$.

Now we give an example of an anti-holomorphic semi-invariant semi-Riemannian submersion.

**Example 1.** *Define $\pi : R^4 \to R_1$ by* $\pi(x_1, x_2, x_3, x_4) = (\frac{x_3 + x_4}{\sqrt{2}})$

*Then the map $\pi$ is a semi-Riemannian submersion and*

$\ker \pi_* = D \oplus D^{\perp}$ *where* $D = span(\partial_1, \partial_2)$

$D^{\perp} = span(\partial_3 + \partial_4)$

*and* $\ker \pi_*^{\perp} = span(\partial_4 - \partial_3)$, *where* $\partial_i = \frac{\partial}{\partial x_i}$

*It is clear from definition the map $\pi$ is a proper anti-holomorphic semi-invariant semi-Riemannian submersion.*

**Lemma** 3.1- Let $\pi$ be a Lagrangian semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, we get

(a) $T_V JX = JT_V X$

(b) $A_\xi JX = JA_\xi X$

where $V$ is a vertical vector field, $\xi$ is a horizontal vector field and $X$ is a vector field on $M$.

It is easy to show that this Lemma holds for an anti-holomorphic semi-invariant semi-Riemannian submersion.

## 4. Integrability and totally Geodesicness

In this section, we shall prove the integrability and totally geodesicness of the distributions.

**Lemma** 4.1- Let $\pi$ be a semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then

(a) The anti-invariant distribution $D^{\perp}$ is always integrable.

(b) The invariant distribution $D$ is always integrable iff

$$g(T_Z JW - T_W JZ, JX) = 0$$

for $Z, W \in D$ and $X \in D^{\perp}$.

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Thus, using Lemma 3.1 and (2.5) we get the following result. From Lemma 4.1 we easily conclude the following result.

**Lemma** 4.2- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then
(a) The anti-invariant distribution $D^\perp$ is always integrable.
(b) The invariant distribution $D$ is always integrable.
Now, we are ready to state one of the main results.

**Theorem** 4.3- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then horizontal distribution $(\ker \pi_*)^\perp$ is integrable and totally geodesic, i.e. $A \equiv 0$.
*Proof.* The proof of this Theorem is similar to the proof of Theorem 4.5 ([7]). So we leave it.

Note that the vertical distribution $\ker \pi_*$ is always integrable.

**Lemma** 4.4- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$.Then the anti-invariant distribution $D^\perp$ defines a totally geodesic foliation in the fibers $\pi^{-1}(x)$, $x \in N$.

Proof. Suppose $X, Y \in D^\perp$ and $Z \in D$
using (2.1), (2.2), (2.7) and Lemma 3.1 we have

$$\begin{aligned} g(\hat{\nabla}_X Y, Z) &= g(\nabla_X Y, Z) \\ &= g(J\nabla_X JY, Z) = g(\nabla_X JY, JZ) \\ &= g(T_X JY, JZ) = g(JT_X Y, JZ) \\ &= -g(T_X Y, Z) = 0 \\ g(\hat{\nabla}_X Y, Z) &= 0 \end{aligned}$$

this complete the proof.

Also in a similar way, we have the following results.

**Lemma** 4.5- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then the anti-invariant distribution $D$ defines a totally geodesic foliation in the fibers $\pi^{-1}(x)$, $x \in N$.

By Lemma 4.4 and 4.5, we have the result.

**Theorem** 4.6-Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then the fibers of $\pi$ are locally product semi-Riemannian manifolds.

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Proof. If we see O'Niell tensor $T$ of the anti-holomorphic semi-invariant submersion $\pi$.

Suppose $U, V \in \ker \pi_*$ and $\xi \in (\ker \pi_*)^\perp$ since $(\ker \pi_*)^\perp = J(D^\perp)$ there is a vector field $X \in D^\perp$ such that $\xi = JX$.

Then, we get

$$\begin{aligned} g(T_U V, \xi) &= g(T_U V, JX) \\ &= -g(JT_U V, X) \\ &= -g(T_U JV, X) \end{aligned}$$

Hence for $V \in D$ we have

$$g(T_U V, \xi) = 0 \tag{4.1}$$

From (4.1) we get

$$T_U D = 0 \tag{4.2}$$

for $U \in \ker \pi_*$. Thus using equation (4.2), we have the following main result.

**Theorem** 4.7- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$.Then, we have

(a) $T_X Z = 0 = T_Z X$

(b) $T_Z W = 0$ where $X \in D^\perp$ and $Z, W \in D$.

We easily see that from Theorem 4.7, $T_Z \xi = 0$ for any $Z \in D$ and $\xi \in (\ker \pi_*)^\perp$.

Thus, we have the following results.

**Corollary** 4.8- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, we have always $T_Z \equiv 0$ for $Z \in D$.

From the part (a) of the Theorem 4.7, we get:

**Corollary** 4.9- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, the fibers of $\pi$ are always mixed totally geodesic.

From the part (b) of Theorem 4.7, we have that:

**Corollary** 4.10- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, the foliation of the invariant distribution $D$ are totally geodesic in the total space $M$.

Also from Theorem 4.7, it follows that.

**Corollary** 4.11- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, $T \equiv 0$ iff $T_X Y = 0$ for all $X, Y \in D^\perp$ i.e. $T_{D^\perp} D^\perp = 0$.

Hence, we can also get the following results.

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**Corollary** 4.12- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then $(\ker \pi_*)$ defines a totally geodesic foliation iff $T_{D^\perp} D^\perp = 0$.

Since O'Niell's tensor $A \equiv 0$ and by Corollary 4.12, we get:

**Theorem** 4.13- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then, $M$ is a locally product manifold $M_{(\ker \pi_*)} \times M_{(\ker \pi_*)^\perp}$ iff $T_{D^\perp} D^\perp = 0$.

## 5. Totally Geodesicness and Harmonicity of the Anti-Holomorphic Semi-Invariant Semi-Riemannian Submersion

The smooth map $\phi$ between two semi-Riemannian manifolds is called totally geodesic if $\nabla \phi_* = 0$

We shall examine the totally geodesicness and harmonicity of an anti-invariant submersion in this section. Here we give necessary and sufficient condition for an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$ to be a totally geodesic map.

**Theorem** 5.1- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Then $\pi$ is a totally geodesic map iff $T_{D^\perp} D^\perp = 0$.

Proof. Since $\pi$ is a semi-Riemannian submersion we have

$$(\nabla \pi_*)(E, F) = 0 \tag{5.1}$$

for all $E, F \in (\ker \pi_*)^\perp$ and for any $X, Y \in \ker \pi_*$,
using (2.7) we have

$$\begin{aligned} (\nabla \pi_*)(X, Y) &= -\pi_*(\nabla_X Y) \\ &= -\pi_*(T_X Y + \hat{\nabla}_X Y) \\ &= -\pi_*(T_X Y) \end{aligned}$$

since $\pi$ is linear isometry between $(\ker \pi_*)^\perp$ and $\Gamma TN$.
Hence, it follows that $(\nabla \pi_*)(X, Y) = 0$ iff $T_X Y = 0$ for all $X, Y \in \ker \pi_*$ that is;

$$(\nabla \pi_*)(X, Y) = 0 \Leftrightarrow T \equiv 0 \tag{5.2}$$

Similarly for any $X \in \ker \pi_*$ and $E \in (\ker \pi_*)^\perp$, using (2.9), we get

$$\begin{aligned} (\nabla \pi_*)(E, X) &= -\pi_*(\nabla_E X) \\ &= -\pi_*(A_E X + v \nabla_E X) \end{aligned}$$

Since $\pi$ is linear isometry between $(\ker \pi_*)^\perp$ and $\Gamma TN$ and $A \equiv 0$, it gives that

$$(\nabla \pi_*)(E, X) = 0 \tag{5.3}$$

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for any $X \in \ker \pi_*$ and $E \in (\ker \pi_*)^\perp$.

Thus from (5.1), (5.2) and (5.3) we have $(\nabla \pi_*) = 0$ iff $T \equiv 0$.

Now, we shall examine the harmonicity of an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$. Recall that a smooth map $\phi$ is harmonic iff it has minimal fibers [19]. Thus the submersion $\pi$ is harmonic iff $\sum_{k=1}^{2m+n} T_{v_k} v_k = 0$

where $(v_1, ........, v_{2m+n})$ is a local orthonormal frame of $(\ker \pi_*)$ but because of Theorem 4.7, it follows that $\pi$ is harmonic iff $\sum_{k=1}^{n} T_{e_i} e_i = 0$.

**Theorem** 5.2- Let $\pi$ be an anti-holomorphic semi-invariant semi-Riemannian submersion from a para-Kähler manifold $(M, g, J)$ onto a semi-Riemannian manifold $(N, g_n)$.Then, $\pi$ is harmonic iff

$$trace JT_X = 0 \text{ for all } X \in D^\perp.$$

Proof. Let $X \neq 0$ vector field in $D^\perp$
Then, for $1 \leq i \leq n$, using the skew symmetricalness of $T_E$
Using Lemma 3.1 and (2.5), we have

$$\begin{aligned} g(T_{e_i} e_i, JX) &= -g(T_{e_i} Je_i, X) \\ &= -g(JT_{e_i} X, e_i) \\ &= -g(JT_X e_i, e_i) \end{aligned}$$

Hence, we get

$$g(\sum_{k=1}^{n} T_{e_i} e_i, JX) = -\sum_{k=1}^{n} g(JT_X e_i, e_i) \quad (5.4)$$

for all $X \in D^\perp$.

Thus from (5.4) we have the results.

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